Dimensional Characteristics of Multimode Diffusion Chaos

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Some parabolic systems of the reaction-diffusion type exhibit the phenomenon of diffusion chaos. Specifically, when the diffusivities decrease proportionally, while the other parameters of a system remain fixed, the system exhibits a chaotic attractor which dimension increases indefinitely. Various finite-dimensional models of diffusion chaos are considered that represent chains of coupled ordinary differential equations and similar chains of discrete mappings. A numerical analysis suggests that these chains with suitably chosen parameters exhibit chaotic attractors of arbitrarily high dimensions.

"Reaction – diffusion" systems are an important class of nonlinear dynamical systems in which spatially inhomogeneous oscillation modes are due to the presence of the diffusion component. Such systems are often encountered in physical, biochemical and population dynamics applications. By diffusion chaos we understand the strange attractor of the system "reaction - diffusion" non-trivially depending on the space variable. Currently, there are two concepts of diffusion chaos: low-mode and multimode chaos. The first of these can occur when diffusion parameter values are average and the second occurs when the diffusion parameter tends to zero.

The interest for low-mode chaos was inspired by the well-known works by E.Lorenz, D.Ruelle and F. Takens and then by Y. Kuramoto. The general question raised there is if it is possible to find a correspondence between stochastic regimes in a distributed dynamical system with infinite-dimensional phase space and strange attractors in finite-dimensional model system. There are many papers describing the situations where the answer is positive.

We consider the growth of the Lyapunov dimension of the attractor of the evolution system as the diffusion coefficient decreases to be one of the features of multimode diffusion chaos. In the work a number of parabolic boundary value problems are analyzed numerically and it shows that the phenomenon occurs. The Ginzburg-Landau equation is considered for different parameter values. We present an extended numerical experiment for generalized Klein-Gordon equation which illustrates the feasibility of Landau-Sell scenario.